A MODIFIED INVERSE-CHEBYSHEV FILTER WITH AN ALL POSITIVE ELEMENTS LADDER PASSIVE REALIZATION

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ABSTRACT
This paper presents a technique to obtain a realization for passive ladder networks of the Inverse-Chebyshev lowpass filter with all positive elements. The technique consists in the shifting of transmission zeros to infinity by means of a transformation in the complex frequency s-plane. The technique allows for multiple frequency shifts until the desired realization is obtained. Results show that there are additional improvements in group delay and in stopband attenuation.

1. INTRODUCTION

Inverse-Chebyshev filters were first developed as an alternative to Chebyshev filters. This is due mainly to the fact that the Inverse-Chebyshev filter characteristic possesses a better group delay response when compared to a Chebyshev filter. Typical group delay responses for both types of filters are shown in Fig. 1. There we can see the improvement achieved by the Inverse-Chebyshev filter over the regular Chebyshev one. A disadvantage of the Inverse-Chebyshev characteristic is that the passive realization requires more elements because it has to realize finite transmission zeros. A fifth order passive ladder filter is shown in Fig. 2. There we can see that the finite transmission zeros have to be realized with LC parallel resonant circuits.

Furthermore, even order transfer functions do not have a zero at infinity as shown in Fig. 3 for a sixth order Inverse Chebyshev function.
The lack of at least a zero at infinity precludes the realization of a passive ladder network with all positive passive elements.

In a previous paper [2], it was shown that a transformation had to be carried on the even order transfer function in order to obtain a passive ladder realization with positive elements. This transformation is given by

$$s^2 = \frac{p^2 (\Omega_x^2 - 1)}{p^2 + \Omega_x^2}$$  \hspace{1cm} (1)

where $p$ denotes either an original pole or zero, $s$ denotes the new position for such a pole (zero), and $\Omega_x$ is the finite zero with the largest magnitude, that is, the zero farthest from the origin. This transformation allows us to shift the largest zero pair to infinity providing thus with at least a zero at infinity (actually, we now have two zeros at infinity.) Applying Eq. (1) to even order transfer functions allows us to obtain a passive ladder realization for even order transfer functions. By using this technique we compiled the realizations for Table 1. A complete version of this table was included in [1]. In this table we see that still there are some realizations missing. The reason is that they had negative valued elements. In order to overcome this problem we apply the same transformation given by Eq.(1) to either odd or even order elements. By doing this we shift additional zeros to infinity leaving a simpler realization for the ladder network because now the function has fewer finite transmission zeros. The resulting transfer function can now be synthesized as a ladder network. In case that still there are negative elements, Eq. (1) can be applied again. Of course the limit is the number of finite zeros that the Inverse-Chebyshev function initially had.

2. SYNTHESIS OF THE LADDER CIRCUIT

The synthesis procedure for the ladder circuit can be started once the appropriate transducer function $N(s)$ has been obtained. Then we have to start obtaining the input impedances needed for the synthesis. The first step in the synthesis procedure is to write the transducer function as $[1,3]$ $$(N(j\omega))^2 = \frac{R_z}{4R_t} \frac{|V_{in}|^2}{|V_{out}|^2}$$  \hspace{1cm} (2)

where $V_{in}$ and $V_{out}$ are the input and output voltages, respectively, in the lossless ladder shown in Fig. 3. Also, $|N(j\omega)|^2$ is related to the characteristic function $K(s)$ by Feldkeller’s equation

$$|N(j\omega)|^2 = 1 + |K(j\omega)|^2$$  \hspace{1cm} (3)

Tabla 1 Inverse-Chebyshev element values for passive ladder realization.

<table>
<thead>
<tr>
<th>n</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$c_4$</th>
<th>$c_5$</th>
<th>$c_6$</th>
<th>$c_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1.17172</td>
<td>2.34344</td>
<td>0.32004</td>
<td>1.17172</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.51237</td>
<td>1.57309</td>
<td>0.52663</td>
<td>1.98949</td>
<td>0.92877</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.06700</td>
<td>0.85352</td>
<td>0.54375</td>
<td>1.44096</td>
<td>0.88341</td>
<td>1.09701</td>
<td>0.71706</td>
</tr>
<tr>
<td>3</td>
<td>1.86644</td>
<td>3.73287</td>
<td>0.20092</td>
<td>1.86644</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.90568</td>
<td>2.41716</td>
<td>0.34273</td>
<td>2.72155</td>
<td>1.21008</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.57338</td>
<td>1.75305</td>
<td>0.19719</td>
<td>2.2791</td>
<td>1.25023</td>
<td>0.72347</td>
<td>0.02010</td>
</tr>
<tr>
<td>6</td>
<td>0.23756</td>
<td>1.16423</td>
<td>0.09865</td>
<td>1.78558</td>
<td>1.37482</td>
<td>0.67515</td>
<td>1.09099</td>
</tr>
</tbody>
</table>
If we write the functions $N(s)$ and $K(s)$ as

$$N(s) = \frac{C_0 E(s)}{P(s)}$$  \hspace{1cm} (4a)

$$K(s) = \frac{F(s)}{P(s)}$$  \hspace{1cm} (4b)

Then, Eq.(3) becomes

$$C_0^2 E(s)E(-s) = P(s)P(-s) + F(s)F(-s)$$  \hspace{1cm} (5)

where $C_0$ is chosen such that the maximum gain in the passband has some predetermined value (usually unity is the most used value for equal terminating resistances). That is, $|N(j\omega)| = 1$. Once we have determined the polynomial $F(s)$ that satisfies Eq.(5), we can use it to obtain the impedance $z_{11}(s)$ needed in the synthesis procedure. This impedance is given by [3]

$$z_{11}(s) = \frac{C_0 E_e(s) + F_e(s)}{C_0 E_o(s) - F_o(s)}$$  \hspace{1cm} (6)

where the subindexes $e$ and $o$ denote even and odd parts, respectively. The synthesis procedure will include partial and full removals of pole pairs and poles at infinity, to produce finite transmission zeros and the zeros at infinity [1]. There is still the possibility that a negative-valued element arises in the synthesis procedure. In that case another zero pair has to be shifted to infinity and the procedure started again.

### 3. EXAMPLE

As an example of the technique described above, let us consider a seventh order transfer function with $\omega_k = 1$ rad/sec, $A_{\text{max}} = 30$dB. The transfer function is given by [1]

$$N(s) = \frac{(s^2 + 1.0257^2)(s^2 + 1.2791^2)(s^2 + 2.3048^2)}{s^7 + s^6 4.5222 + s^5 10.201 + s^4 14.9889 + s^3 15.3636 + s^2 11.5989 + s 5.7115 + 2.0289}$$

Applying a synthesis technique to obtain a passive ladder realization will result in a network having negative element values, and therefore, non-realizable. By realizing two shiftings of the largest zero-pairs to infinity, similar to what we did for the even order transfer functions we can obtain the new transfer function

$$N(s) = \frac{(s^2 + 1.0704^2)}{s^7 + s^6 3.1401 + s^5 4.9302 + s^4 4.9733 + s^3 3.4672 + s^2 1.6675 + s 0.5088 + 0.07417}$$

whose magnitude is shown in Fig. 4. In this plot we can see the original inverse-Chebyshev function. We can see that the new function has only a transmission zero pair while the original one has the three pairs in addition to the zero at infinity. Moreover, we can see that the new function has improved the stopband attenuation. For completeness, the group delay of the modified transfer function is shown in Fig. 5 together with the group delay of the original Inverse Chebyshev function.

For the synthesis procedure we have to calculate the new function $F(s)$ which from Eq.(5) we obtain as (the value of $C_0$ is 15.4514)

$$F(s) = 15.4514 s^7 + 42.5972 s^6 + 58.7168 s^5 + 51.4208 s^4 + 30.2549 s^3 + 11.4767 s^2 + 2.2188 s$$

This function is to be used together with $E(s)$ in Eq. (6) to obtain $z_{11}(s)$ as

$$z_{11}(s) = \frac{5.9225 s^6 + 25.4243 s^4 + 14.2886 s^2 + 1.1459}{30.9029 s^7 + 134.8962 s^5 + 83.8284 s^3 + 10.0726 s}$$

which renders the all-positive elements passive ladder realization shown in Fig. 6.
4. CONCLUSIONS

We have presented a technique to obtain a passive realization for Inverse-Chebyshev filter characteristic having all-positive elements. This is achieved by shifting some of the zeros to infinity by means of a spectral transformation. Thus, the resulting transfer function, and therefore, the ladder passive realization, will be less complex. In addition to this, the resulting characteristic has a better group delay response and improved stopband attenuation.

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Fig. 4. Magnitude responses for modified inverse-Chebyshev filter and the regular Inverse Chebyshev magnitude.

Fig. 5. Group delay for the original and modified Inverse-Chebyshev filter.

Fig. 6. Passive ladder realization for the modified Inverse-Chebyshev function.

6. REFERENCES


